Question Bank PT-1

$$y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$$

Given: $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$$

$$= 2\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \cdot \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$$

$$= 2\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \cdot \left(\frac{1}{2\sqrt{x}} - \frac{2}{\sqrt{x^3}}\right)$$

$$= 2\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}} - \frac{2}{x\sqrt{x}}\right)$$

$$= 2\left(\frac{\sqrt{x}}{2\sqrt{x}} - \frac{2\sqrt{x}}{x\sqrt{x}} + \frac{1}{2x} - \frac{2}{x^2}\right)$$

$$= 2\left(\frac{1}{2} - \frac{2}{x} + \frac{1}{2x} - \frac{2}{x^2}\right)$$

$$= 1 - \frac{4}{x} + \frac{1}{x} - \frac{4}{x^2}$$

$$\frac{dy}{dx} = 1 - \frac{3}{x} - \frac{4}{x^2}$$

5 $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2+\sqrt{x}}{2-\sqrt{x}}\right)$

$$= \frac{(2-\sqrt{x})\frac{d}{dx}(2+\sqrt{x}) - (2+\sqrt{x})\frac{d}{dx}(2-\sqrt{x})}{(2-\sqrt{x})^2}$$

$$= (2-\sqrt{x})\left(\frac{+1}{2\sqrt{x}}\right) - (2+\sqrt{x})\left(\frac{-1}{2\sqrt{x}}\right) \Big/ (2-\sqrt{x})^2$$

$$= \frac{\frac{1}{\sqrt{x}} - \frac{1}{2} + \frac{1}{\sqrt{x}} + \frac{1}{2}}{(2-\sqrt{x})^2}$$

$$= \frac{\frac{2}{\sqrt{x}}}{(2-\sqrt{x})^2}$$

$$= \frac{2}{\sqrt{x}(2-\sqrt{x})^2}$$

$y = e^{\sin 2x} + (\log 6x)^2 + \sqrt{\cos(x/2)}$

differentiate w. r. t x

$\frac{dy}{dx} = \frac{d}{dx}(e^{\sin 2x}) + \frac{d}{dx}(\log 6x)^2 + \frac{d}{dx}(\sqrt{\cos(x/2)})$

$= e^{\sin 2x} \cdot \frac{d}{dx}(\sin 2x) + 2\log 6x \frac{d}{dx}(\log 6x) + \frac{1}{2\sqrt{\cos(x/2)}} \frac{d}{dx}\cos(x/2)$

$= e^{\sin 2x} \cos 2x (2) + 2\log 6x \frac{1}{6x}(6) + \frac{1}{2\sqrt{\cos(x/2)}}(-\sin(x/2))(\frac{1}{2})$

$\frac{dy}{dx} = 2\cos 2x\, e^{\sin 2x} + \frac{12\log 6x}{6x} - \frac{\sin(x/2)}{4\sqrt{\cos(x/2)}}$

$y = \frac{1}{\sin^2 3x \cos^2 3x} = \operatorname{cosec}^2 3x \cdot \sec^2 3x$

$\frac{dy}{dx} = \frac{d}{dx}(\operatorname{cosec}^2 3x \cdot \sec^2 3x)$

$= \operatorname{cosec}^2 3x \frac{d}{dx}(\sec^2 3x) + \sec^2 3x \frac{d}{dx}(\operatorname{cosec}^2 3x)$

$= \operatorname{cosec}^2 3x \cdot 2\sec(3x) \frac{d}{dx}(\sec 3x) + \sec^2 3x \cdot 2\operatorname{cosec} 3x \frac{d}{dx}(\operatorname{cosec} 3x)$

$= \operatorname{cosec}^2 3x\, 2\sec(3x) \sec 3x \cdot \tan 3x (3) + 3\sec^2 3x\, 2\operatorname{cosec} 3x (-\operatorname{cosec} 3x \cot 3x)$

$= 6\operatorname{cosec}^2 3x \cdot \sec^2 3x \cdot \tan 3x - 6\sec^2 3x \operatorname{cosec}^2 3x \cdot \cot 3x$

$\frac{dy}{dx} = 6\operatorname{cosec}^2 3x \cdot \sec^2 3x (\tan 3x - \cot 3x)$

$= 6\operatorname{cosec}^2 3x \cdot \sec^2 3x \left(\frac{\sin 3x}{\cos 3x} - \frac{\cos 3x}{\sin 3x}\right)$

$= 6\operatorname{cosec}^2 3x \cdot \sec^2 3x \left(\frac{\sin^2 3x - \cos^2 3x}{\cos 3x \cdot \sin 3x}\right)$

4 $y = \frac{\cos 2x + \sin 2x}{\cos 2x - \sin 2x}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\cos 2x + \sin 2x}{\cos 2x - \sin 2x}\right)$$

$$= \frac{(\cos 2x - \sin 2x)\frac{d}{dx}(\cos 2x + \sin 2x) - (\cos 2x + \sin 2x)\frac{d}{dx}(\cos 2x - \sin 2x)}{(\cos 2x - \sin 2x)^2}$$

$$= \frac{(\cos 2x - \sin 2x)\left(-\sin 2x \frac{d}{dx}(2x) + \cos 2x \frac{d}{dx}(2x)\right) - (\cos 2x + \sin 2x)\left(-\sin 2x \frac{d}{dx}(2x) - \cos 2x \frac{d}{dx}(2x)\right)}{(\cos 2x - \sin 2x)^2}$$

$$= \frac{2(\cos 2x - \sin 2x)(-\sin 2x + \cos 2x) + 2(\cos 2x + \sin 2x)(\sin 2x + \cos 2x)}{(\cos 2x - \sin 2x)^2}$$

$$= \cancel{2(\cos^2 2x - \sin^2 2x}$$

$$= \frac{2(\cos 2x - \sin 2x)^2 + 2(\cos 2x + \sin 2x)^2}{(\cos 2x - \sin 2x)^2}$$

$$= \frac{2(\cos(2x) - \sin(2x))^2 + (\cos 2x + \sin 2x)^2}{(\cos 2x - \sin 2x)^2}$$

$$= \frac{2(\cos 2x - \sin 2x)^2}{(\cos 2x - \sin 2x)^2} + \frac{2(\cos 2x + \sin 2x)^2}{(\cos 2x - \sin 2x)^2}$$

$$= 2\left(1 + \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x}{\cos^2 x + \sin^2 x - 2\cos x \sin x}\right)$$

$$= 2\left(1 + \frac{1 + \sin 2x}{1 - \sin 2x}\right)$$

$$= 2\left(\frac{1 - \sin 2x + 1 + \sin 2x}{1 - \sin 2x}\right)$$

$$= \frac{4}{1 - \sin 2x}$$

(7) Differentiate $\log(x\sin x)$ w.r.t $\frac{1}{x}$

$y = \log(x\sin x)$

$$\frac{dy}{dx} = \frac{d}{dx}(\log(x\sin x))$$

$$= \frac{1}{x\sin x}\left(x\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x\right)$$

$$= \frac{1}{x\sin x}\left(x\cos x + \sin x\right)$$

$$= \cot x + \frac{1}{x}$$

$$z = \frac{1}{x}$$

$$\frac{dz}{dx} = \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= \frac{-1}{x^2}$$

$$\frac{dy}{dz} = \frac{\frac{x\cot x + 1}{x}}{\frac{-1}{x^2}}$$

$$= \frac{x\cot x + 1}{x} \times \frac{x^2}{-1}$$

$$= -x(x\cot x + 1)$$

Page No.
Date

(5) $y = \frac{2+\sqrt{x}}{}$

(6) $y = \tan^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

$= \tan^{-1}\left(\sqrt{\frac{1+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{1-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}}\right)$

$= \tan^{-1}\left(\sqrt{\frac{\cancel{2}\cos^2\frac{x}{2}}{\cancel{2}\sin^2\frac{x}{2}}}\right)$

$= \tan^{-1}\left(\sqrt{\cot^2\frac{x}{2}}\right)$

$\tan^{-1}\left(\cot\frac{x}{2}\right)$

$\tan^{-1}\left(\tan\left(\pi-\frac{x}{2}\right)\right)$

$\pi - \frac{x}{2}$

differentiate w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}\left(\pi - \frac{x}{2}\right)$

$= 0 - \frac{1}{2}$

$\frac{dy}{dx} = \frac{-1}{2}$

$z = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$= \sin^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$

$= \sin^{-1}\left(2\cos\theta\sin\theta\right)$

$= \sin^{-1}\left(\sin 2\theta\right)$

$= 2\theta$

$= 2\cos^{-1}x$

$= 2\left(\frac{-1}{\sqrt{1-x^2}}\right)$

$= \underline{1}$

(10) $\cancel{\tan^{-1}\left(2x\right.}$

(11) $\cancel{y = e^{\sin 2x}}$

(12) $\cancel{x = \frac{2t}{1+t^2}}$ & y.

Page No
Date

(8) $y = \cancel{x+1}\ x^{\sin x}$

$\frac{dy}{dx} = \sin(x) x^{\sin x + 1} \frac{d}{dx}(\sin x)$

$= \cos x \sin x \cdot x^{\sin x + 1}$

$z = e^{\cos x}$

$\frac{dz}{dx} = e^{\cos x} \frac{d}{dx}(\cos x)$

$= e^{\cos x}(-\sin x)$

$= -\sin x\, e^{\cos x}$

$\frac{dy}{dz} = \frac{\cos x \sin x\, x^{\sin x + 1}}{-\sin x\, e^{\cos x}}$

$= -\cos x\, x^{\sin x + 1} e^{-\cos x}$

(7) $\frac{dy}{dz} = \frac{d}{dx}(\cos^{-1}(2x^2 - 1))$

$= \frac{d}{dx}(\cos^{-1} 2(\cos^2 x - 1))$

$= \cancel{\frac{d}{dx}(\cos^{-1}(2 \cdot 1 - \sin^2 x - 1)}$

$= \frac{d}{dx}(\cos^{-1}(2 - 2\sin^2 x$

$\cancel{d}\ \cos^{-1}(\cancel{2}\cos 2\theta)$

$= 2\theta$

$= \cos^{-1} x$

$\frac{dy}{dx} = \frac{d}{dx}(\cos^{-1} x)$

$= 2\left(\frac{-1}{\sqrt{1 - x^2}}\right)$